12.5, number 33: **Find the point on the line**

$$\begin{cases} x = 4t \\ y = -2t \\ z = 2t \end{cases}$$

which is closest to the point $(0, 0, 12)$**.**

Answer: We find the plane through $(0, 0, 12)$ which is perpendicular to $\overrightarrow{\boldsymbol{v}} = 4\overrightarrow{\boldsymbol{i}} - 2\overrightarrow{\boldsymbol{j}} + 2\overrightarrow{\boldsymbol{k}}$. This plane is

$$4(x - 0) - 2(y - 0) + 2(z - 12) = 0$$

$$4x - 2y + 2z = 24$$

$$2x - y + z = 12$$

The answer is the intersection of the original line and the plane we just created. These objects intersect when

$$2(4t) - (-2t) + (2t) = 12$$

$$12t = 12$$

$$t = 1$$

At $t = 1$, the line hits the point $\boxed{(4, -2, 2)}$.

Check.

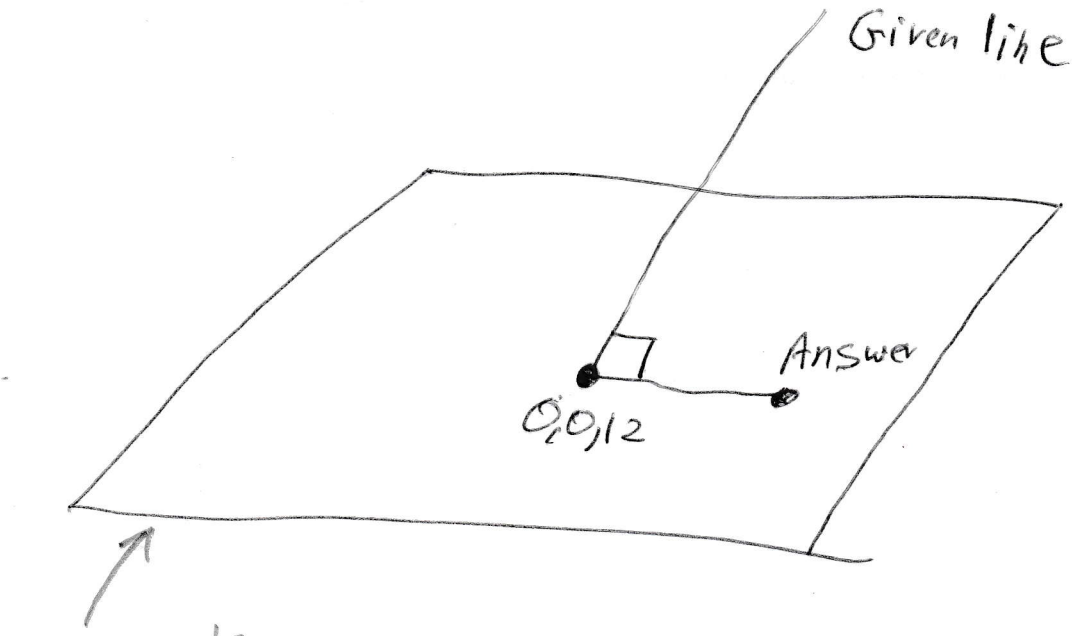
It is clear that the proposed answer is on the line.

We verify that the vector $\overrightarrow{(0, 0, 12)(4, -2, 2)}$ is perpendicular to the line. We compute

$$(4\overrightarrow{\boldsymbol{i}} - 2\overrightarrow{\boldsymbol{j}} - 10\overrightarrow{\boldsymbol{k}}) \cdot (4\overrightarrow{\boldsymbol{i}} - 2\overrightarrow{\boldsymbol{j}} + 2\overrightarrow{\boldsymbol{k}}) = 16 + 4 - 20 = 0\checkmark.$$

There is a picture on the next page.

Picture for 12.5 Number 33



we made this plane to find the answer.