12.5, number 23: **Find the equation of the plane through the points** $P = (1, 1, -1)$**,** $Q = (2, 0, 2)$**, and** $R = (0, -2, 1)$**.**

Answer: The vector $\overrightarrow{PQ} \times \overrightarrow{PR}$ is perpendicular to the plane. The vector $\overrightarrow{PQ} \times \overrightarrow{PR}$ is

$$\begin{vmatrix} \vec{\boldsymbol{i}} & \vec{\boldsymbol{j}} & \vec{\boldsymbol{k}} \\ 1 & -1 & 3 \\ -1 & -3 & 2 \end{vmatrix} = \begin{vmatrix} -1 & 3 \\ -3 & 2 \end{vmatrix} \vec{\boldsymbol{i}} - \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} \vec{\boldsymbol{j}} + \begin{vmatrix} 1 & -1 \\ -1 & -3 \end{vmatrix} \vec{\boldsymbol{k}} = 7\,\vec{\boldsymbol{i}} - 5\,\vec{\boldsymbol{j}} - 4\,\vec{\boldsymbol{k}}$$

The point $P = (x, y, z)$ is on the plane precisely when

$$\overrightarrow{P_0P} \cdot (\overrightarrow{PQ} \times \overrightarrow{PR}) = 0$$

$$((x-1)\,\vec{\boldsymbol{i}} + (y-1)\,\vec{\boldsymbol{j}} + (z+1)\,\vec{\boldsymbol{k}}) \cdot (7\,\vec{\boldsymbol{i}} - 5\,\vec{\boldsymbol{j}} - 4\,\vec{\boldsymbol{k}}) = 0$$

$$7(x-1) - 5(y-1) - 4(z+1) = 0.$$

$$\boxed{7x - 5y - 4z = 6}$$

Check:

P satisfies the proposed answer:

$$7(1) - 5(1) - 4(-1) = 6\checkmark.$$

Q satisfies the proposed answer:

$$7(2) - 5(0) - 4(2) = 6\checkmark.$$

R satisfies the proposed answer:

$$7(0) - 5(-2) - 4(1) = 6\checkmark.$$