12.4, number 35: Find the area of the parallelogram with vertices A = (1, 0), B = (0, 1), C = (-1, 0), and D = (0, -1).

Answer:

We drew a picture on the next page. The parallelogram is actually a square and each side has length $\sqrt{2}$; so the area is 2. We can use cross product to calculate the area. The parallelogram is determined by the vectors \overrightarrow{AB} and \overrightarrow{AD} ; so the area is

$$\begin{split} |\overrightarrow{AB} \times \overrightarrow{AD}| &= |\begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ -1 & 1 & 0 \\ -1 & -1 & 0 \end{vmatrix}| = |\begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix} \overrightarrow{i} - \begin{vmatrix} -1 & 0 \\ -1 & 0 \end{vmatrix} \overrightarrow{j} + \begin{vmatrix} -1 & 1 \\ -1 & -1 \end{vmatrix} \overrightarrow{k} \\ &= |0 \overrightarrow{i} + 0 \overrightarrow{j} + 2 \overrightarrow{k}| = \boxed{2}, \end{split}$$

as we expected.

Picture for 12.4 #35

