12.4, number 15: **Let** $P = (1, -1, 2)$**,** $Q = (2, 0, -1)$**, and** $R = (0, 2, 1)$**. Find the area of the triangle determined by the points** $P$**,** $Q$**, and** $R$**. Also find a unit vector perpendicular to the plane containing** $P$**,** $Q$**, and** $R$**.**

**Answer:** The area of the triangle determined by $P$, $Q$, $R$ is $\frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|$ and a unit vector perpendicular to the plane containing $P$, $Q$, and $R$ is $\frac{1}{|\overrightarrow{PQ}\times\overrightarrow{PR}|}\overrightarrow{PQ} \times \overrightarrow{PR}$. (Of course, $\overrightarrow{PQ} \times \overrightarrow{PR}$ can be replaced by $\overrightarrow{QP} \times \overrightarrow{QR}$ or $\overrightarrow{RQ} \times \overrightarrow{RP}$. The sign of the cross product might change; but the basic direction and the length will not change.)

We compute

$$\begin{aligned}\overrightarrow{PQ} \times \overrightarrow{PR} &= \begin{vmatrix} \overrightarrow{\boldsymbol{i}} & \overrightarrow{\boldsymbol{j}} & \overrightarrow{\boldsymbol{k}} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -3 \\ 3 & -1 \end{vmatrix} \overrightarrow{\boldsymbol{i}} - \begin{vmatrix} 1 & -3 \\ -1 & -1 \end{vmatrix} \overrightarrow{\boldsymbol{j}} + \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} \overrightarrow{\boldsymbol{k}} \\ &= 8\overrightarrow{\boldsymbol{i}} + 4\overrightarrow{\boldsymbol{j}} + 4\overrightarrow{\boldsymbol{k}}\end{aligned}$$

The area of the triangle is $\boxed{(\frac{1}{2})4\sqrt{6}}$ and a unit vector perpendicular to the plane containing the triangle is

$$\boxed{\frac{1}{4\sqrt{6}}(8\overrightarrow{\boldsymbol{i}} + 4\overrightarrow{\boldsymbol{j}} + 4\overrightarrow{\boldsymbol{k}})}.$$