12.3, number 1abd: **Let $\overrightarrow{\boldsymbol{v}} = 2\overrightarrow{\boldsymbol{i}} - 4\overrightarrow{\boldsymbol{j}} + \sqrt{5}\overrightarrow{\boldsymbol{k}}$ and $\overrightarrow{\boldsymbol{u}} = -2\overrightarrow{\boldsymbol{i}} + 4\overrightarrow{\boldsymbol{j}} - \sqrt{5}\overrightarrow{\boldsymbol{k}}$. Find $\overrightarrow{\boldsymbol{v}} \cdot \overrightarrow{\boldsymbol{u}}$, $|\overrightarrow{\boldsymbol{v}}|$, $|\overrightarrow{\boldsymbol{u}}|$, the cosine of the angle between $\overrightarrow{\boldsymbol{v}}$ and $\overrightarrow{\boldsymbol{u}}$, and the projection of $\overrightarrow{\boldsymbol{u}}$ onto $\overrightarrow{\boldsymbol{v}}$.**

Answer:

$$\begin{aligned}\overrightarrow{\boldsymbol{v}} \cdot \overrightarrow{\boldsymbol{u}} &= (2\overrightarrow{\boldsymbol{i}} - 4\overrightarrow{\boldsymbol{j}} + \sqrt{5}\overrightarrow{\boldsymbol{k}}) \cdot (-2\overrightarrow{\boldsymbol{i}} + 4\overrightarrow{\boldsymbol{j}} - \sqrt{5}\overrightarrow{\boldsymbol{k}}) \\ &= 2(-2) + (-4)(4) + \sqrt{5}(-\sqrt{5}) = -4 - 16 - 5 = \boxed{-25}.\end{aligned}$$

$$|\overrightarrow{\boldsymbol{v}}| = \sqrt{(2)^2 + (-4)^2 + (\sqrt{5})^2} = \sqrt{4 + 16 + 5} = \sqrt{25} = \boxed{5}.$$

$$|\overrightarrow{\boldsymbol{u}}| = \sqrt{(-2)^2 + (4)^2 + (-\sqrt{5})^2} = \sqrt{4 + 16 + 5} = \boxed{5}.$$

We know that $\overrightarrow{\boldsymbol{v}} \cdot \overrightarrow{\boldsymbol{u}} = |\overrightarrow{\boldsymbol{v}}||\overrightarrow{\boldsymbol{u}}| \cos\theta$, where θ is the angel between $\overrightarrow{\boldsymbol{v}}$ and $\overrightarrow{\boldsymbol{u}}$. So,

$$-25 = 5(5)\cos\theta$$

and $\boxed{-1 = \cos\theta}$.

Of course this means that the angle between $\overrightarrow{\boldsymbol{v}}$ and $\overrightarrow{\boldsymbol{u}}$ is π radians (or 180 degrees). In other words $\overrightarrow{\boldsymbol{v}}$ and $\overrightarrow{\boldsymbol{u}}$ point in opposite directions. Of course, our answer to the last part of the problem will exactly confirm this.

$$\text{proj}_{\overrightarrow{\boldsymbol{v}}}\, \overrightarrow{\boldsymbol{u}} = \frac{\overrightarrow{\boldsymbol{u}} \cdot \overrightarrow{\boldsymbol{v}}}{\overrightarrow{\boldsymbol{v}} \cdot \overrightarrow{\boldsymbol{v}}}\overrightarrow{\boldsymbol{v}} = \frac{-25}{25}\overrightarrow{\boldsymbol{v}} = -\overrightarrow{\boldsymbol{v}}$$

$$\boxed{\text{The projection of } \overrightarrow{\boldsymbol{u}} \text{ onto } \overrightarrow{\boldsymbol{v}} \text{ is } -\overrightarrow{\boldsymbol{v}}.}$$