f

20. True or False. If true, **prove** it. If false, then give a **counterexample**. The sum of any three consecutive integers is divisible by 3 .

True Three cases.

① If $n=3k$, then $n+(n+1)+(n+2)$
$= 3k+(3k+1)+(3k+2) = 9k+3 = 3(3k+1)$

② If $n=3k+1$, then $n+(n+1)+(n+2) =$
$(3k+1)+(3k+2)+(3k+3) = 9k+6 = 3(3k+2)$

③ If $n=3k+2$, then $n+(n+1)+(n+2) =$
$(3k+2)+(3k+3)+(3k+4) = 9k+9 = 3(3k+3)$

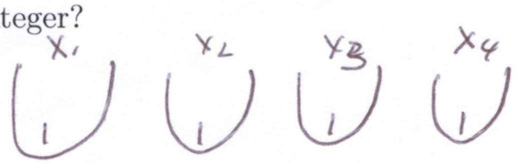
21. True or False. If true, **prove** it. If false, then give a **counterexample**. If a sum of two integers is even, then one of the summands is even. (In the expression $a+b$, a and b are called *summands*.)

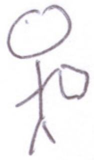
False $1+1=2$ and 2 is even

1 is odd

1 or more

22. How many solutions does $x_1 + x_2 + x_3 + x_4 = 20$ have, if each x_i is a positive integer?





16 Drops 3 switches $\binom{16+3}{3}$ possible work orders.

$\binom{19}{3} = \frac{19 \cdot \cancel{18}^{3} \cdot 17}{\cancel{6}}$

$\binom{20+3}{3} \Leftarrow 3$ $\binom{23}{4}$

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