f

16. Prove that

$$\left(1-\frac{1}{2^2}\right)\cdot\left(1-\frac{1}{3^2}\right)\cdot\ldots\cdot\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n},$$

for all integers $n \geq 2$.

when $n=2$ LHS $= 1-\frac{1}{4}=\frac{3}{4}$ RHS $=\frac{3}{4}$ ✓

Assume IH $\left(1-\frac{1}{2^2}\right)\cdot\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$

We must prove $\left(1-\frac{1}{2^2}\right)\cdot\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{n^2}\right)\cdot\left(1-\frac{1}{(n+1)^2}\right)=\frac{n+2}{2n+2}$ *

LHS of * $= \frac{n+1}{2n}\cdot\left(1-\frac{1}{(n+1)^2}\right)=\frac{n+1}{2n}\left(\frac{n^2+2n+1-1}{(n+1)^2}\right)=\frac{\cancel{(n+1)}(n^2+2n)}{2n(n+1)^{\cancel{2}}}$

↑
use IH
here

$=\frac{\cancel{n}(n+2)}{2\cancel{n}(n+1)}=\frac{n+2}{2(n+1)}$ which equals the right side of *.

17. True or False. If true, **prove** it. If false, then give a **counterexample**. If an integer is a perfect square, then its cube root is irrational.

False 64 is a perfect square and $\sqrt[3]{64}=4$ which is rational.

18. True or False. If true, **prove** it. If false, then give a **counterexample**. The difference of any two irrational numbers is irrational.

False $\sqrt{2}$ and $\sqrt{2}$ are both irrational but $\sqrt{2}-\sqrt{2}=0$ which is rational

19. Find integers q and r so that $56=5q+r$ with $0 \leq r < 5$.

$56 = 5\cdot 11 + 1$

$q=11$
$r=1$