Final Math 174 Fall 1998

PRINT Your Name:_____

There are 25 problems on 7 pages. Each problem is worth 6 points.

CIRCLE your answers. **No Calculators.**

1. Is the following argument valid?
   All honest people pay their taxes.
   Darth is not honest.
   ∴ Darth does not pay his taxes.

   P X is honest
   Q X pays taxes

   $P \to Q$
   $\sim P$
   $\therefore \sim Q$
   inverse error (Not Valid)

2. What is the negation of "The sum of any two irrational numbers is irrational"?

   There exists a pair of irrational numbers whose sum is rational.

3. Give an example of a function f from the set of integers to the set of integers which is onto, but not one-to-one.

   $$f(n) = \begin{cases} n & \text{if } n \le 0 \\ n-1 & \text{if } 1 \le n \end{cases}$$

   f is not one-to-one because $f(0) = f(1) = 0$
   f is onto because if $n \le 0$ then $f(n) = n$ and if $1 \le n$, then $f(n+1) = n$.

4. True or False. If true, **prove** it. If false, then give a **counterexample**. If $f: X \to Y$ and $g: Y \to Z$ are functions, with $g \circ f$ onto, then g is onto.

   (True) $g \circ f$ is onto so if $z \in Z$, then $\exists x \in X$ with $(g \circ f)(x) = z$
   $\therefore g(f(x)) = z$ and $z \in$ the image of g.

5. What is the coefficient of x^5 in $(2x+3)^8$?

   $$(2x+3)^8 = \sum_{k=0}^{8} \binom{8}{k} (2x)^k 3^{8-k}.$$

   The coefficient of x^5 is $\binom{8}{5}(2)^5(3)^3$.

   $\frac{8 \cdot 7 \cdot 6}{6} \cdot 32 \cdot 27$

   48384