e4

11. Prove

$$\binom{5}{0} + \binom{6}{1} + \binom{7}{2} + \cdots + \binom{5+n}{n} = \binom{6+n}{n}$$

for all integers n with $0 \leq n$.

Proof by induction.

When $n=0$ The formula is $\binom{5}{0} = \binom{6+0}{0}$ This is true

Assume $\sum_{k=0}^{n} \binom{5+k}{k} = \binom{6+n}{n}$. This is IH

Prove $\sum_{s=0}^{n+1} \binom{5+s}{s} = \binom{7+n}{n+1}$ $*$

The LHS of $* = \sum_{s=0}^{n} \binom{5+s}{s} + \binom{5+n+1}{n+1} = \binom{6+n}{n} + \binom{6+n}{n+1} = \binom{7+n}{n+1}$, which

Use IH here

This is the identity from Pascal's triangle

is the right side of $*$.

12. What is the coefficient of x^4 in $(3x+2)^9$?

The relevant term is $\binom{9}{4}(3x)^4 2^5$

So the coefficient is $\binom{9}{4} 3^4 2^5$