e4

5. Prove $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

By induction

when $n=1$ the equation is $1 = \frac{1(1+1)(2(1)+1)}{6}$ and this is true

Assume $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ holds for some fixed n. This is IH.

Prove $\displaystyle\sum_{k=1}^{n+1} k^2 = \frac{(n+1)(n+2)(2n+3)}{6}$ (*)

The LHS of * $= \displaystyle\sum_{k=1}^{n} k^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2 =$

use IH

$= \frac{n+1}{6}\left[n(2n+1) + 6(n+1)\right] = \frac{n+1}{6}\left[2n^2 + 7n + 6\right] = \frac{n+1}{6}(n+2)(2n+3)$

which is the right side of *

6. (9 points) A coin is tossed 10 times. What is the probability that exactly 5 of the toses will land as heads?

There are 2^{10} possible outcomes _ _ _ _ _ _ _ _ _ _

(Fill in each blank with T or H)

There are $\binom{10}{5}$ ways to select 5 positions for H

$$\frac{\binom{10}{5}}{2^{10}}$$