e3

4. In the world series the first team to win four games in a row wins the series. How many ways can a world series be played if no team wins two games in a row?

Suppose the two teams are called N and A. There are two ways

either N wins first NANANAN

or A wins first ANANANA

5. True or False. If true, **prove** it. If false, then give a **counterexample**. If n is an integer with $1 \le n$, then

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n\cdot(n+1)}=\frac{n}{n+1}.$$

True Proof by induction.

When $n=1$ the proposed identity is $\frac{1}{1\cdot 2}=\frac{1}{1+1}$ which is true

Induction Hypothesis Assume $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n\cdot(n+1)}=\frac{n}{n+1}$

We must show that $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}+\frac{1}{(n+1)(n+2)}=\frac{n+1}{n+2}$ ⊛

The left side of ⊛ is $\left(\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}\right)+\frac{1}{(n+1)(n+2)}\underset{\uparrow}{=}\frac{n}{n+1}+\frac{1}{(n+1)(n+2)}$

Use IH here

$=\frac{1}{n+1}\left[n+\frac{1}{n+2}\right]=\frac{1}{n+1}\left[\frac{n(n+2)+1}{n+2}\right]=\frac{n^2+2n+1}{(n+1)(n+2)}=\frac{(n+1)^2}{(n+1)(n+2)}=\frac{n+1}{n+2}$)

which is the right side of ⊛. The proof is complete.