e2

7. True or False. If true, **prove it**. If false, then give a **counterexample**. If $p_1, p_2, p_3, \ldots, p_r$ are prime integers, then $N = p_1 p_2 p_3 \cdots p_r + 1$ is a prime integer.

False

Take $p_1 = 3$ with $r=1$, then $N = 4$ which is not prime

or take $p_1 = 3$ $p_2 = 5$ with $r=1$, then $N = 16$ which is not prime

or Take $p_1 = 2$, $p_2 = 3$ $p_3 = 5$ $p_4 = 7$ $p_5 = 11$ $p_6 = 13$ with $r = 6$,

then $N = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 + 1 = 30031 = 59 \cdot 509$ which is not prime

8. True or False. If true, **prove it**. If false, then give a **counterexample**. The number $\sqrt{3}$ is irrational.

True Proof By Contradiction

Assume $\sqrt{3}$ is rational.

Then $\sqrt{3} = \frac{a}{b}$ where a and b are integers with no common factors

So $b\sqrt{3} = a$

So $b^2 3 = a^2$

3 is prime with $3 | a^2$ $\therefore 3 | a$

$\therefore a = 3a'$

$\therefore b^2 3 = (3a')^2 = 9(a')^2$

$\therefore b^2 = 3(a')^2$

But 3 is prime and $3 | b^2$

$\therefore 3 | b$

Thus a and b have no common factor but 3 is a common factor of a and b

This is a contradiction. Our original assumption is false.

Thus $\sqrt{3}$ is irrational.