Math 174, Fall 2003, Solution to Quiz 9

Problem: Consider the Tower of Hanoi problem. There are three towers in a row: tower A, tower B, and tower C. There are n disks of different sizes stacked on tower A. One must move all n disks to tower C. One may NEVER place a bigger disk on top of a smaller disk. In the present problem, one may move a disk only to an ADJACENT tower. Let a_n be the minimum number of moves needed to transfer a stack of n disks from tower A to tower C. Find a_1, a_2, a_3. Find a recurrence relation for $a_1, a_2, a_3, \dots$.

Answer: It is clear that $\boxed{a_1 = 2}$. Move the disk to tower B. Move the disk to tower C. For a_2: move the small disk to tower C (2 moves). Move the big disk to tower B (one move). Move the small disk back to tower A (two moves). Move the big disk to tower C (one move). Move the small disk back to tower C (2 moves). So, $\boxed{a_2 = 8}$. For a_3: move the two small disks to Tower C (a_2 moves). Move the big disk to tower B (1 move). Move the two small disks to tower A (a_2 moves). Move the big disk to tower C (1 move). Move the two small disks back to tower C (a_2 moves). So, $a_3 = 3a_2 + 2 = \boxed{26}$. Of course, we now see how to do the general problem. Move the $n-1$ small disks to Tower C (a_{n-1} moves). Move the big disk to tower B (1 move). Move the $n-1$ small disks to tower A (a_{n-1} moves). Move the big disk to tower C (1 move). Move the $n-1$ small disks back to tower C (a_{n-1} moves). So, $\boxed{a_n = 3a_{n-1} + 2}$.