Math 174, Fall 2003, Solution to Quiz 3

Question: Prove that the square of any integer has the form $3k$ or $3k+1$ for some integer k .

Answer: Let n be an arbitrary integer. There are three cases: $n=3\ell$, $n=3\ell+1$, or $n=3\ell+2$ for some integer ℓ .

In the first case, $n=3\ell$, so $n^2=9\ell^2=3(3\ell^2)$, which has the form $3k$, with $k=3\ell^2$.

In the second case, $n=3\ell+1$, so $n^2=9\ell^2+6\ell+1=3(3\ell^2+2\ell)+1$, which has the form $3k+1$, with $k=3\ell^2+2\ell$.

In the third case, $n=3\ell+2$, so $n^2=9\ell^2+12\ell+4=3(3\ell^2+4\ell+1)+1$, which has the form $3k+1$, with $k=3\ell^2+4\ell+1$.