**Math 142, Final exam, Spring, 2004**

PRINT Your Name:______________________________

There are 20 problems on 10 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS! CHECK** your answer whenever possible.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**. Otherwise, get your grade from VIP.

I will post the solutions on my website when the exam is finished.

1. Find $\displaystyle\int \sin^2 x \cos^3 x\, dx$ . Check your answer.

2. Find $\displaystyle\int_{-3}^{0} \frac{1}{(x+1)^2}\, dx$ . Check your answer.

3. Find $\displaystyle\int x \sin x\, dx$ . Check your answer.

4. Find $\displaystyle\int x \sin(x^2)\, dx$ . Check your answer.

5. Find the area of the region bounded by $y = \ln x$ , the $x$ -axis, and $x = 2$ .

6. Let $f(x) = e^{-x^2}$ . Where is $f(x)$ increasing, decreasing, concave up, and concave down? Find the local maxima, local minima, and points of inflection of $y = f(x)$ . Find the horizontal asymptotes of $y = f(x)$ . Graph $y = f(x)$ .

7. Find $\lim\limits_{x \to 0} \frac{\sin(x^2) - x^2 + \frac{x^6}{6}}{x^{10}}$ . Justify your answer.

8. Find $\displaystyle\int \frac{dx}{\sqrt{x^2-1}}$ . Check your answer.

9. Find $\displaystyle\int \frac{3x^2+4x+4}{x^3+4x} dx$ . Check your answer.

10. Where does the function
$$f(x) = \sum_{n=1}^{\infty} \frac{(x+3)^n}{2^n \cdot n^2}$$
converge? Justify your answer.

11. What familiar function is equal to
$$f(x) = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \frac{x^8}{9!} - \dots ?$$
Justify your answer.

12. Find the limit of the sequence whose $n^{\text{th}}$ term is $a_n = \left(1 - \dfrac{1}{3n}\right)^n$. Justify your answer.

13. Find $\dfrac{d}{dx}(x2^x)$.

14. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$ converge or diverge? Justify your answer.

15. Approximate $e^{\frac{-1}{10}}$ with an error at most $\frac{1}{1000}$. Justify your answer.

16. Find the Taylor polynomial $P_3(x)$ for $f(x) = \ln x$ about $c = 1$.

17. Take $P_3(x)$ and $f(x)$ from problem 16. Estimate the error that is introduced if $f(x)$ is approximated by $P_3(x)$ for $.9 \le x \le 1.1$. Justify your answer.

18. Suppose that the government pumps an extra \$1 billion into the economy. Assume that each business and individual saves 20% of its income and spends the rest, so that of the initial \$1 billion, 80% is respent by individuals and businesses. Of that amount, 80% is spent, and so forth. What is the total increase in spending due to the government action? Justify your answer.

19. Money is invested at an annual interest rate of 4% compounded continuously. How much money should be invested today in order for the investment to be worth \$100,000 twenty years from today? Justify your answer.

20. Use Trapezoidal rule, with $n = 4$, to approximate $\int_0^1 \frac{1}{1+x^2}dx$. Recall that Trapezoidal rule says that

$$\int_a^b f(x)dx = \frac{h}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right] + E_n,$$

for $h = \frac{b-a}{n}$, $x_i = a+hi$, and $E_n = -\frac{(b-a)^3}{12n^2}f''(c)$ for some $c$ with $a \le c \le b$. (Just record the sum. You are not required to add the fractions. You are not required to do anything with $E_n$.)