Math 142, Spring 2004, Exam 4

PRINT Your Name:__________________________________

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

I won't grade your exam until Monday. So don't be surprised if I don't e-mail your grade to you until then.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will post the solutions on my website on **Monday**.

1. A ball is dropped from the height of 30 feet. Each time it hits the floor it rebounds to $\frac{5}{6}$ its previous height. Find the total distance it travels. Explain what you are doing.

2. Give a closed formula for $s_n = \sum\limits_{k=2}^{n} \frac{1}{k-1} - \frac{1}{k+1}$. (Your formula should be exactly equal to the sum I have given. Your formula should not contain any dots or any summation signs.) Explain what you are doing.

3. Approximate $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^4}$ with an error of at most $\frac{1}{500}$. Be sure to explain what you are doing and why you are allowed to do it.

4. Give an upper bound for the difference between $\sum\limits_{n=1}^{5} \frac{1}{n^3}$ and $\sum\limits_{n=1}^{\infty} \frac{1}{n^3}$. I expect your upper bound to be relatively small and correct. Be sure to explain what you are doing and why you are allowed to do it.

5. Does the series $\sum\limits_{n=1}^{\infty} \frac{\sin n}{n^3}$ converge or diverge? Justify your answer.

6. Does the series $\sum\limits_{n=1}^{\infty} \left(\frac{n-1}{n}\right)^n$ converge or diverge? Justify your answer.

7. Does the series $\sum\limits_{n=1}^{\infty} \left(\frac{-3}{4}\right)^n$ converge or diverge? Justify your answer.

8. Does the series $\sum\limits_{n=1}^{\infty} \frac{2^n}{n!}$ converge or diverge? Justify your answer.

9. Where does the function
$$f(x) = \frac{x-1}{1} + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + \frac{(x-1)^4}{4} + \dots$$
converge? Justify your answer.

10. What familiar function is equal to
$$f(x) = \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$$
for $-1 < x < 1$? Justify your answer.