PRINT Your Name:____________________

Quiz – September 14, 2006

Find the length of the curve

$$\begin{cases} x = \frac{1}{3}t^3 \\ y = \frac{1}{2}t^2, \end{cases}$$

for $0 \le t \le 1$.

Answer: The arc length is equal to

$$\int_{\text{beginning}}^{\text{end}} ds = \int_{\text{beginning}}^{\text{end}} \sqrt{dx^2 + dy^2} = \int_{\text{beginning}}^{\text{end}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^1 \sqrt{(t^2)^2 + (t)^2}\, dt = \int_0^1 \sqrt{t^4 + t^2}\, dt = \int_0^1 t\sqrt{t^2 + 1}\, dt = \frac{1}{2}\frac{2}{3}(t^2 + 1)^{\frac{3}{2}}\Big|_0^1$$

$$= \frac{1}{2}\frac{2}{3}(2^{\frac{3}{2}} - 1) = \boxed{\frac{1}{3}(2^{\frac{3}{2}} - 1)}$$