PRINT Your Name:____________________

Quiz – September 10, 2004

The population of the United States was 3.9 million in 1790 and 178 million in 1960. If the rate of growth is assumed proportional to the number present, what estimate would you give for the population in 2000? (You may leave ln in your answer.)

Answer: Let $P(t)$ equal the population at time t, where t is measured in tears, with $t = 0$ taken to be 1790. We are told that $P(0) = 3.9 \times 10^6$ and $P(170) = 178 \times 10^6$. We want to find $P(210)$. The words which come between "If" and the comma tell us that $\frac{dP}{dt} = kP$ for some constant k. We solved this differential equation in class and learned that $P(t) = P(0)e^{kt}$. Plug in $t = 170$ to see that $178 \times 10^6 = 3.9 \times 10^6 e^{170k}$. In other words, $\frac{178}{3.9} = e^{170k}$ and $\ln\left(\frac{178}{3.9}\right) = 170k$; hence, $\frac{1}{170}\ln\left(\frac{178}{3.9}\right) = k$. Now we know that

$$P(t) = 3.9(10^6)e^{\frac{t}{170}\ln\left(\frac{178}{3.9}\right)}$$

for all times t. In particular,

$$\boxed{P(210) = 3.9(10^6)e^{\frac{210}{170}\ln\left(\frac{178}{3.9}\right)}.}$$

If you are sitting at home with a calculator in your hand, you compute

$$3.9(10^6)e^{\frac{210}{170}\ln\left(\frac{178}{3.9}\right)} \approx 437.3 \times 10^6.$$

We notice that this number is significantly higher than the actual US population in 2000. Can you think of any cultural happenings that occurred about 1960 which caused a big change in the birth rate in the United States?