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Quiz – August 31, 2006

Find $\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^x$.

Answer: There are two ways to do this problem.

Method 1: Maneuver the problem into a problem that you have already done and write down the answer. You might know that

$$(*) \qquad \lim_{x\to\infty}\left(1+\frac{r}{x}\right)^x = e^r.$$

If so, then the given problem is (*) with $r=\frac{1}{2}$ and the answer is $\boxed{\sqrt{e}}$.

Method 2: Use L'Hopitals rule. Let $y=\left(1+\frac{1}{2x}\right)^x$. We want to find $\lim_{x\to\infty} y$. We can find

$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty} x\ln(1+\tfrac{1}{2x}) = \lim_{x\to\infty}\frac{\ln(1+\frac{1}{2x})}{\frac{1}{x}}.$$

The top and the bottom both go to 0, so L'Hopitals rule tells us that the last expression is equal to

$$\lim_{x\to\infty}\frac{\frac{\frac{-1}{2x^2}}{(1+\frac{1}{2x})}}{\frac{-1}{x^2}} = \lim_{x\to\infty}\frac{1}{2(1+\frac{1}{2x})} = \frac{1}{2}.$$

Thus, the answer is

$$\lim_{x\to\infty} y = \lim_{x\to\infty} e^{\ln y} = \boxed{e^{1/2}}.$$