PRINT Your Name:____________________

Quiz – August 31, 2004

Find the volume of the solid generated by revolving the region bounded by $y = e^x$, $y = 0$, $x = 0$, and $x = \ln 3$ about the x-axis.

Answer: Draw the picture. Our plan is to approximate the region by using discs. Chop the x-axis from $x = 0$ to $x = \ln 3$ into small pieces. Over each piece draw a rectangle. Spin each rectangle. Get a disc (see the picture) of volume $\pi r^2 t$, where t is the thickness (for us this is dx which is a little piece of the x-axis) and r is the radius (for us this is the y-coordinate at the top of our rectangle minus the y-coordinate at the bottom of our rectangle, all written in terms of x; in other words: e^x). So each disc has volume $\pi(e^x)^2 dx = \pi e^{2x} dx$. We add up the volume inside all of the discs and take the limit. This amounts to finding the definite integral:

$$\pi \int_0^{\ln 3} e^{2x} dx = \pi \frac{e^{2x}}{2}\Big|_0^{\ln 3} = \frac{\pi}{2}(e^{2\ln 3} - 1) = \frac{\pi}{2}(9 - 1) = \boxed{4\pi.}$$