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Quiz – August 27, 2004

Let $f(x) = 4x^2$ for $x \le 0$.

(a) Find $f^{-1}(x)$.

(b) What is the domain of $f^{-1}(x)$?

(c) Verify that $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}.

Answer: Let $y = f^{-1}(x)$. So $f(y) = x$. (Notice, for future reference, that y is in the domain of f, so $y \le 0$.) At any rate, we have $4y^2 = x$ and we want to find y. Divide both sides by 4 to get $y^2 = \frac{1}{4}x$. Take the square root of both sides to get $y = \pm\frac{1}{2}\sqrt{x}$. We aren't finished because the we have not found a **function** for $f^{-1}(x)$. However, we saw above that $y \le 0$; so $y = -\frac{1}{2}\sqrt{x}$. Our answer to (a) is $\boxed{f^{-1}(x) = -\dfrac{1}{2}\sqrt{x}}$. Our answer to (b) is $\boxed{\text{all } x \ge 0}$; because, we can insert any number which is non-negative into $\sqrt{x}$. For part (c), we start with $x \ge 0$. We observe that

$$f(f^{-1}(x)) = f(-\frac{1}{2}\sqrt{x}) = 4(-\frac{1}{2}\sqrt{x})^2 = 4\frac{1}{4}(\sqrt{x})^2 = x. \checkmark$$