PRINT Your Name:____________________________________

Quiz – April 1, 2004

Does the series

$$\frac{2}{1\cdot 3\cdot 4}+\frac{3}{2\cdot 4\cdot 5}+\frac{4}{3\cdot 5\cdot 6}+\frac{5}{4\cdot 6\cdot 7}+\cdots$$

converge? Justify your answer.

Answer: The series is $\sum\limits_{k=2}^{\infty}\frac{k}{(k-1)(k+1)(k+2)}$. We apply the limit comparison test to $\sum\limits_{k=2}^{\infty}\frac{1}{k^2}$. We see that the limit of the quotient of corresponding terms is

$$\lim_{k\to\infty}\frac{a_k}{b_k}=\lim_{k\to\infty}\frac{\frac{k}{(k-1)(k+1)(k+2)}}{\frac{1}{k^2}}=\lim_{k\to\infty}\frac{k^3}{(k-1)(k+1)(k+2)}$$

Divide top and bottom by k^3 to get $\lim\limits_{k\to\infty}\frac{1}{(1-\frac{1}{k})(1+\frac{1}{k})(1+\frac{2}{k})}=1$. We know that 1 is a number with $1\neq 0$ and $1\neq\infty$. We conclude that $\sum\limits_{k=2}^{\infty}\frac{k}{(k-1)(k+1)(k+2)}$ and $\sum\limits_{k=2}^{\infty}\frac{1}{k^2}$ both converge or both diverge. On the other hand, $\sum\limits_{k=2}^{\infty}\frac{1}{k^2}$ is the p-series with $p=2>1$; thus, $\sum\limits_{k=2}^{\infty}\frac{1}{k^2}$ converges; hence, $\sum\limits_{k=2}^{\infty}\frac{k}{(k-1)(k+1)(k+2)}$ also converges.