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Quiz – March 2, 2004

Find
$$\int \frac{2x^2+x-8}{x^3+4x}dx.$$

Answer: Set
$$\frac{2x^2+x-8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}.$$

Multiply both sides by $x(x^2+4)$ to get

$$2x^2+x-8 = A(x^2+4)+(Bx+C)x.$$

That is,
$$2x^2+x-8 = (A+B)x^2+Cx+4A.$$

Equate the corresponding coefficients to get:

$$\begin{aligned} 2 &= A+B \\ 1 &= C \\ -8 &= 4A. \end{aligned}$$

The bottom equation says $A=-2$. The middle equation says $C=1$. The top equation says $B=4$. We check what we have so far:

$$\frac{-2}{x}+\frac{4x+1}{x^2+4} = \frac{-2(x^2+4)+(4x+1)x}{x(x^2+4)} = \frac{2x^2+x-8}{x(x^2+4)}.\checkmark$$

The original problem is

$$\int \left(\frac{-2}{x}+\frac{4x+1}{x^2+4}\right)dx = \boxed{-2\ln|x|+2\ln(x^2+4)+\frac{1}{2}\arctan\left(\frac{x}{2}\right)+C.}$$

By the way, the derivative of $\frac{1}{2}\arctan(\frac{x}{2})$ is

$$\frac{1}{2}\frac{\frac{1}{2}}{1+\left(\frac{x}{2}\right)^2} = \frac{1}{4\left(1+\frac{x^2}{4}\right)} = \frac{1}{4+x^2},$$

as expected.