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Quiz – February 5, 2004

Find

$$\int \frac{t^2 \cos^2(t^3-2)}{\sin^2(t^3-2)}\,dt.$$

Check your answer.

Answer: Let $u = t^3 - 2$. So, $du = 3t^2\,dt$, and $\frac{1}{3}du = t^2\,dt$. The original integral is equal to

$$\frac{1}{3}\int \frac{\cos^2 u}{\sin^2 u}\,du = \frac{1}{3}\int \cot^2 u\,du.$$

Recall that $\sin^2 u + \cos^2 u = 1$. Divide both sides by $\sin^2 u$ to see that $1 + \cot^2 u = \csc^2 u$. The original problem is

$$= \frac{1}{3}\int (\csc^2 u - 1)\,du = \frac{1}{3}(-\cot u - u) + C = \boxed{-\frac{1}{3}(\cot(t^3-2) + (t^3-2)) + C.}$$

Check: The derivative of the proposed answer is

$$-\frac{1}{3}\left[3t^2(-\csc^2(t^3-2) + 3t^2\right] = t^2(\csc^2(t^3-2) - 1) = t^2\cot^2(t^3-2).\checkmark$$