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Quiz – February 24, 2004

1. (5 points) Find

$$\int \frac{3x-13}{x^2+3x-10}\,dx.$$

Answer: We see that $x^2+3x-10=(x+5)(x-2)$. We set

$$\frac{3x-13}{x^2+3x-10}=\frac{A}{x+5}+\frac{B}{x-2}.$$

Multiply both sides by $(x+5)(x-2)$ to see that

$$3x-13=A(x-2)+B(x+5).$$

So

$$3x-13=(A+B)x+(-2A+5B).$$

Equate the corresponding coefficients to see that

$$\begin{cases} 3=A+B \\ -13=-2A+5B \end{cases}$$

Replace Equation 2 by equation 2 plus two copies of equation 1 to get:

$$\begin{cases} 3=A+B \\ -7=\quad +7B \end{cases}$$

So $B=-1$ and $A=4$. By the way

$$\frac{4}{x+5}+\frac{-1}{x-2}=\frac{4(x-2)-(x+5)}{(x+5)(x-2)}=\frac{3x-13}{(x+5)(x-2)},$$

as expected. So the original integral is equal to

$$\int \frac{4}{x+5}+\frac{-1}{x-2}\,dx=\boxed{4\ln|x+5|-\ln|x-2|+C}.$$

2. (5 points) Find $\int \frac{1}{\sqrt{x^2+4}}\,dx$.

Answer: We do a Trig substitution. Let $x=2\tan\theta$. It follows that $dx=2\sec^2\theta\,d\theta$,

$$\sqrt{x^2+4}=\sqrt{4\tan^2\theta+4}=\sqrt{4(\tan^2 x+1)}=\sqrt{4\sec^2\theta}=2\sec\theta,$$

and the integral is

$$\int \tfrac{2\sec^2\theta}{2\sec\theta}\,d\theta=\int\sec\theta\,d\theta=\ln|\sec\theta+\tan\theta|+C=\boxed{\ln|\tfrac{\sqrt{x^2+4}}{2}+\tfrac{x}{2}|+C.}$$

Check: The derivative of the proposed answer is

$$\frac{\frac{2x}{4\sqrt{x^2+4}}+\frac{1}{2}}{\frac{\sqrt{x^2+4}}{2}+\frac{x}{2}}=\frac{\frac{x}{\sqrt{x^2+4}}+1}{\sqrt{x^2+4}+x}=\frac{\left(\frac{x}{\sqrt{x^2+4}}+1\right)\sqrt{x^2+4}}{\left(\sqrt{x^2+4}+x\right)\sqrt{x^2+4}}=\frac{x+\sqrt{x^2+4}}{\left(\sqrt{x^2+4}+x\right)\sqrt{x^2+4}}$$

$$=\frac{1}{\sqrt{x^2+4}}.\checkmark$$