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Quiz – February 10, 2004

Find

$$\int \sin^5 4x \cos^2 4x \, dx.$$

Check your answer.

Answer: The original problem is equal to

$$\int \sin 4x (1 - \cos^2 4x)^2 \cos^2 4x \, dx.$$

Let $u = \cos 4x$. So, $du = -4 \sin 4x \, dx$. The original integral is equal to

$$-\frac{1}{4} \int (1 - u^2)^2 u^2 \, du = -\frac{1}{4} \int (1 - 2u^2 + u^4) u^2 \, du = -\frac{1}{4} \int (u^2 - 2u^4 + u^6) \, du$$

$$= -\frac{1}{4} \left(\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7} \right) + C = \boxed{-\frac{1}{4} \left(\frac{\cos^3 4x}{3} - \frac{2 \cos^5 4x}{5} + \frac{\cos^7 4x}{7} \right) + C.}$$

Check: The derivative of the proposed answer is

$$-\frac{1}{4} \left(\cos^2 4x (-4 \sin 4x) - 2 \cos^4 4x (-4 \sin 4x) + \cos^6 4x (-4 \sin 4x) \right)$$

$$= -\frac{-4 \sin 4x}{4} \cos^2 4x \left(1 - 2 \cos^2 4x + \cos^4 4x \right)$$

$$= \sin 4x \cos^2 4x \left(1 - \cos^2 4x \right)^2$$

$$= \sin 4x \cos^2 4x \left(\sin^2 4x \right)^2 = \sin^5 4x \cos^2 4x. \checkmark$$