PRINT Your Name:____________________

Quiz 9 — March 16, 2012 – Section 7 – 11:15 – 12:05

Remove everything from your desk except a pencil or pen.

Write in complete sentences.
The quiz is worth 5 points.

Consider the series $\sum\limits_{n=1}^{\infty}(e^{1/n}-e^{1/(n+1)})$.

1. Find a closed formula for the N^{th} partial sum $s_N=\sum\limits_{n=1}^{N}(e^{1/n}-e^{1/(n+1)})$. **Please notice that N and n are different!**
2. Does the series $\sum\limits_{n=1}^{\infty}(e^{1/n}-e^{1/(n+1)})$ converge? Explain your answer very thoroughly.
3. Find the sum of the series $\sum\limits_{n=1}^{\infty}(e^{1/n}-e^{1/(n+1)})$, if possible. Explain your answer very thoroughly.

Answer:

1. We see that

$$s_N=\sum_{n=1}^{N}(e^{1/n}-e^{1/(n+1)})=(e^{1/1}-e^{1/2})+(e^{1/2}-e^{1/3})+\cdots+(e^{1/N}-e^{1/(N+1)})$$

$$=e-e^{1/(N+1)}.$$

2. and 3. We compute $\lim\limits_{N\to\infty}s_N=\lim\limits_{N\to\infty}(e-e^{1/(N+1)})=e-e^0=e-1$. We conclude that the series $\sum\limits_{n=1}^{\infty}(e^{1/n}-e^{1/(n+1)})$ converges to $e-1$.