PRINT Your Name:____________________

Quiz 7 — October 2, 2009 – 8:00 section

Remove everything from your desk except this page and a pencil or pen.

$\boxed{\text{Circle}}$ your answer. **Show your work.** Check your answer!
The quiz is worth 5 points.

Compute $\displaystyle\int \frac{dx}{2x^2+4x+7}$.

Answer: Complete the square: $2x^2+4x+7 = 2(x^2+2x+\boxed{1})+7-\boxed{2} = 2(x+1)^2+5$.
Let $\sqrt{2}(x+1) = \sqrt{5}\tan\theta$. It follows that $\sqrt{2}dx = \sqrt{5}\sec^2\theta d\theta$.We compute

$$2(x+1)^2+5 = 5\tan^2\theta+5 = 5(\tan^2\theta+1) = 5\sec^2\theta.$$

The integral is

$$\int \frac{\frac{\sqrt{5}\sec^2\theta d\theta}{\sqrt{2}}}{5\sec^2\theta} = \frac{1}{\sqrt{10}}\int d\theta = \frac{1}{\sqrt{10}}\theta + C = \boxed{\frac{1}{\sqrt{10}}\arctan\left(\frac{\sqrt{2}(x+1)}{\sqrt{5}}\right)+C}.$$

Check. The derivative of the proposed answer is

$$\left(\frac{1}{\sqrt{10}}\right)\left(\frac{\sqrt{2}}{\sqrt{5}}\right)\left(\frac{1}{1+\left(\frac{\sqrt{2}(x+1)}{\sqrt{5}}\right)^2}\right) = \left(\frac{1}{5}\right)\left(\frac{1}{1+\left(\frac{\sqrt{2}(x+1)}{\sqrt{5}}\right)^2}\right)$$

$$= \left(\frac{1}{5+\left(\sqrt{2}(x+1)\right)^2}\right) = \left(\frac{1}{5+(2(x^2+2x+1))}\right) = \frac{1}{2x^2+4x+7}. \checkmark$$