PRINT Your Name:____________________

Quiz 6 — September 25, 2009 – 8:00 section

Remove everything from your desk except this page and a pencil or pen.

$\boxed{\text{Circle}}$ your answer. **Show your work.**
The quiz is worth 5 points.

A particle moving along the x-axis has velocity function $v(t) = t^3 \sin t$. How far does the particle travel from time $t = 0$ to $t = \pi$.

Answer: The velocity is always positive; so the distance traveled is

$$\int_0^{\pi} t^3 \sin t dt.$$

We apply integration by parts: $\int u dv = uv - \int v du$ three times. In the first application, let $u = t^3$ and $dv = \sin t dt$. It follows that $du = 3t^2 dt$ and $v = -\cos t$. Thus the distance traveled is

$$= \left[-t^3 \cos t + \int 3t^2 \cos t dt\right]_0^{\pi}.$$

Now let $u = t^2$ and $dv = \cos t dt$; so $du = 2t dt$ and $v = \sin t$. The distance traveled is

$$= \left[-t^3 \cos t + 3\left(t^2 \sin t - 2\int t \sin t dt\right)\right]_0^{\pi}.$$

Now let $u = t$ and $dv = \sin t dt$; so $du = dt$ and $v = -\cos t$. The distance traveled is

$$\begin{aligned} &= \left[-t^3 \cos t + 3t^2 \sin t - 6\left(-t \cos t + \int \cos t dt\right)\right]_0^{\pi} \\ &= \left[-t^3 \cos t + 3t^2 \sin t + 6t \cos t - 6 \sin t\right]_0^{\pi} \\ &= \boxed{\pi^3 - 6\pi}. \end{aligned}$$

Check: The derivative of

$$-t^3 \cos t + 3t^2 \sin t + 6t \cos t - 6 \sin t$$

is

$$t^3 \sin t - 3t^2 \cos t + 3t^2 \cos t + 6t \sin t - 6t \sin t + 6 \cos t - 6 \cos t = t^3 \sin t,$$

as expected. ✓