PRINT Your Name:____________________

Quiz 4 — February 7, 2014 – Section 8 – 10:50 – 11:40

Remove everything from your desk except this page and a pencil or pen.
The solution will be posted soon after the quiz is given.

$\boxed{\text{Circle}}$ your answer. **Show your work. Your work must be correct and coherent. Check your answer.**
The quiz is worth 5 points.

Find $\int \sqrt{1-4x^2}dx$.

Answer: We let $2x = \sin\theta$. It follows that $2dx = \cos\theta d\theta$ and
$1-4x^2 = 1-\sin^2\theta = \cos^2\theta$. The original problem is

$$\int \sqrt{1-4x^2}dx = \tfrac{1}{2}\int \cos^2\theta d\theta = \tfrac{1}{4}\int (1+\cos 2\theta)d\theta$$

$$= \tfrac{1}{4}(\theta + \tfrac{1}{2}\sin 2\theta) + C = \tfrac{1}{4}(\theta + \sin\theta\cos\theta) + C$$

$$= \tfrac{1}{4}\left(\arcsin(2x) + 2x\sqrt{1-4x^2}\right) + C = \boxed{\tfrac{1}{4}\arcsin(2x) + \tfrac{1}{2}x\sqrt{1-4x^2} + C}.$$

Check. The derivative of the proposed answer is

$$\frac{1}{4}\frac{2}{\sqrt{1-4x^2}} + \frac{1}{2}\left[x\frac{-8x}{2\sqrt{1-4x^2}} + \sqrt{1-4x^2}\right] = \frac{1}{2}\left[\frac{1}{\sqrt{1-4x^2}} - \frac{4x^2}{\sqrt{1-4x^2}} + \sqrt{1-4x^2}\right]$$

$$= \frac{1}{2}\left[\frac{1-4x^2}{\sqrt{1-4x^2}} + \sqrt{1-4x^2}\right] = \frac{1}{2}\left[\sqrt{1-4x^2} + \sqrt{1-4x^2}\right] = \frac{1}{2}\left[2\sqrt{1-4x^2}\right]$$

$$= \sqrt{1-4x^2}. \checkmark$$