PRINT Your Name:___________________

Quiz 3 — January 31, 2014 – Section 7 – 12:00 – 12:50

Remove everything from your desk except this page and a pencil or pen.
The solution will be posted soon after the quiz is given.

Circle your answer. **Show your work. Your work must be correct and coherent.**
The quiz is worth 5 points.

Find $\int \sqrt{x^2+2x}dx$.

Answer: We complete the square: $x^2+2x=(x^2+2x+1)-1=(x+1)^2-1$. Let $x+1=\sec\theta$. It follows that $(x+1)^2-1=\tan^2\theta$ and $dx=\sec\theta\tan\theta d\theta$. The original problem is equal to

$$\int \tan^2\theta\sec\theta d\theta.$$

We use integration by parts. Let $u=\tan\theta$ and $dv=\sec\theta\tan\theta d\theta$. It follows that $du=\sec^2\theta d\theta$ and $v=\sec\theta$. So

$$\int \tan^2\theta\sec\theta d\theta=\sec\theta\tan\theta-\int\sec^3\theta d\theta=\sec\theta\tan\theta-\int(\tan^2\theta+1)\sec\theta d\theta$$

$$=\sec\theta\tan\theta-\int\sec\theta d\theta-\int\tan^2\theta\sec\theta d\theta.$$

Add $\int\tan^2\theta\sec\theta d\theta$ to both sides to see that

$$2\int\tan^2\theta\sec\theta d\theta=\sec\theta\tan\theta-\int\sec\theta d\theta.$$

So

$$\int\sqrt{x^2+2x}dx=\int\tan^2\theta\sec\theta d\theta=(1/2)\left[\sec\theta\tan\theta-\int\sec\theta d\theta\right]$$

$$=(1/2)\left[\sec\theta\tan\theta-\ln|\sec\theta+\tan\theta|\right]+C$$

$$=\boxed{(1/2)\left[(x+1)\sqrt{x^2+2x}-\ln|(x+1)+\sqrt{x^2+2x}|\right]+C}.$$

Check. The derivative of

$$(1/2)\left[(x+1)\sqrt{x^2+2x}-\ln[(x+1)+\sqrt{x^2+2x}\right]$$

is

$$(1/2)\left[\frac{(x+1)(2x+2)}{2\sqrt{x^2+2x}}+\sqrt{x^2+2x}-\frac{1+\frac{2x+2}{2\sqrt{x^2+2x}}}{(x+1)+\sqrt{x^2+2x}}\right]$$

$$=(1/2)\left[\frac{(x+1)^2}{\sqrt{x^2+2x}}+\sqrt{x^2+2x}-\frac{1+\frac{x+1}{\sqrt{x^2+2x}}}{(x+1)+\sqrt{x^2+2x}}\right]$$

$$= (1/2)\left[\frac{(x+1)^2}{\sqrt{x^2+2x}} + \sqrt{x^2+2x} - \frac{\sqrt{x^2+2x}+x+1}{[(x+1)+\sqrt{x^2+2x}]\sqrt{x^2+2x}}\right]$$

$$= (1/2)\left[\frac{(x+1)^2}{\sqrt{x^2+2x}} + \sqrt{x^2+2x} - \frac{1}{\sqrt{x^2+2x}}\right]$$

$$= \frac{1}{2\sqrt{x^2+2x}}\left[(x+1)^2 + x^2 + 2x - 1\right]$$

$$= \frac{1}{2\sqrt{x^2+2x}}\left[2x^2 + 4x\right]$$

$$= \frac{1}{\sqrt{x^2+2x}}\left[x^2 + 2x\right] = \sqrt{x^2+2x}. \checkmark$$