PRINT Your Name:____________________

Quiz 3 — September 16, 2012 – Section 2 – 4:40 – 5:30

Remove everything from your desk except this page and a pencil or pen.
The solution will be posted soon after the quiz is given.

$\boxed{\text{Circle}}$ your answer. **Show your work. Your work must be correct and coherent. Check your answer.**
The quiz is worth 5 points.

Find $\int \frac{1}{(x+5)^2(x-1)}dx$.

Answer: We apply the technique of partial fractions. We must find constants A, B, C so that

$$\frac{1}{(x+5)^2(x-1)} = \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{x-1}.$$

Multiply both sides by $(x+5)^2(x-1)$. We must solve

$$1 = A(x+5)(x-1) + B(x-1) + C(x+5)^2$$

$$1 = \begin{cases} Ax^2 & +4Ax & -5A \\ & +Bx & -B \\ +Cx^2 & +10Cx & +25C \end{cases}$$

We must find constants A, B, and C so that

$$1 = (A+C)x^2 + (4A+B+10C)x + (25C-5A-B).$$

Equate the corresponding coefficients. We must solve

$$\begin{aligned} 0 &= A+C \\ 0 &= 4A+B+10C \\ 1 &= -5A-B+25C \end{aligned}$$

The top equation gives $C = -A$. The middle equation becomes $0 = -6A + B$. The bottom equation becomes $1 = -30A - B$. Add the two most recent equations to see $1 = -36A$. Thus,

$$A = \frac{-1}{36}, \quad B = \frac{-6}{36}, \quad \text{and} \quad C = \frac{1}{36}.$$

We have shown that

$$\frac{1}{(x+5)^2(x-1)} = \frac{1}{36}\left(\frac{-1}{x+5} + \frac{-6}{(x+5)^2} + \frac{1}{x-1}\right).$$

By the way this is correct. The right hand side is

$$\frac{-(x+5)(x-1)-6(x-1)+(x+5)^2}{36(x+5)^2(x-1)} = \frac{-(x+5)(x-1)-6(x-1)+(x+5)^2}{36(x+5)^2(x-1)}$$

$$= \frac{1}{(x+5)^2(x-1)}.$$

We now have

$$\int \frac{1}{(x+5)^2(x-1)} dx = \frac{1}{36} \int \left(\frac{-1}{x+5} + \frac{-6}{(x+5)^2} + \frac{1}{x-1} \right) dx$$

$$= \boxed{\tfrac{1}{36} \left(-\ln|x+5| + \tfrac{6}{x+5} + \ln|x-1| \right) + C.}$$