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Quiz – March 30, 2006

Does the series $\sum_{k=1}^{\infty} \frac{5^k+k}{k!+3}$ converge? **Explain very thoroughly.**

Answer: Notice that

$$\frac{5^k+k}{k!+3} < \frac{5^k+5^k}{k!} = \frac{2\cdot 5^k}{k!}$$

because the fraction on the right has a larger numerator and a smaller denominator. The series $\sum_{k=1}^{\infty} \frac{2\cdot 5^k}{k!}$ converges by the ratio test:

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{\frac{2\cdot 5^{k+1}}{(k+1)!}}{\frac{2\cdot 5^k}{k!}} = \lim_{k\to\infty} \frac{2\cdot 5^{k+1}}{(k+1)!}\frac{k!}{2\cdot 5^k} = \lim_{k\to\infty} \frac{5}{k+1} = 0 < 1.$$

Both series $\sum_{k=1}^{\infty} \frac{5^k+k}{k!+3}$ and $\sum_{k=1}^{\infty} \frac{2\cdot 5^k}{k!}$ are positive series. The terms of $\sum_{k=1}^{\infty} \frac{5^k+k}{k!+3}$ are smaller than the terms of $\sum_{k=1}^{\infty} \frac{2\cdot 5^k}{k!}$. The series $\sum_{k=1}^{\infty} \frac{2\cdot 5^k}{k!}$ converges. We apply the Comparison test to conclude that the series $\sum_{k=1}^{\infty} \frac{5^k+k}{k!+3}$ also converges.