PRINT Your Name:____________________

Quiz – March 23, 2006

Consider the series

$$e^{-x}+e^{-2x}+e^{-3x}+e^{-4x}+e^{-5x}+\dots\ .$$

Find all values of x for which the series converges and find the sum of the series for those values of x. **Explain thoroughly.**

Answer: The series is a geometric series with initial term $a=e^{-x}$ and ratio $r=e^{-x}$. If $-1<e^{-x}<1$, then the series converges to

$$\frac{a}{1-r}=\frac{e^{-x}}{1-e^{-x}}=\frac{1}{e^x-1}.$$

Of course, e^{-x} is always positive; so, the only constraint is that $\frac{1}{e^x}<1$. This is the same as saying $1<e^x$, and this is the same as $0<x$. We conclude that

$$\boxed{\text{if } 0<x, \text{ then the series converges to } \frac{1}{e^x-1}.}$$