PRINT Your Name:____________________

Quiz – February 16, 2006

Find $\int \frac{dx}{(4+x^2)^2}$. **Check your answer.**

Answer: Let $x = 2\tan\theta$. It follows that $dx = 2\sec^2\theta d\theta$, $4 + x^2 = 4\sec^2\theta$, and the integral is equal to

$$\int \frac{2\sec^2\theta d\theta}{16\sec^4\theta} = \frac{1}{8}\int \frac{d\theta}{\sec^2\theta} = \frac{1}{8}\int \cos^2\theta d\theta = \frac{1}{16}\int (1+\cos 2\theta)d\theta$$

$$= \frac{1}{16}(\theta + \frac{\sin 2\theta}{2}) + C = \frac{1}{16}(\theta + \frac{2\sin\theta\cos\theta}{2}) + C.$$

Draw a triangle with the side opposite the angle θ of length x, the adjacent side is 2, the hypothenuse is $\sqrt{x^2+4}$. We now see that $\sin\theta = \frac{x}{\sqrt{x^2+4}}$ and $\cos\theta = \frac{2}{\sqrt{x^2+4}}$. We know know that the integral is equal to:

$$\boxed{\frac{1}{16}\left(\arctan\frac{x}{2} + \frac{2x}{x^2+4}\right) + C.}$$

Check: The derivative of the proposed answer is

$$\frac{1}{16}\left(\frac{\frac{1}{2}}{1+\frac{x^2}{4}} - 2x\frac{2x}{(x^2+4)^2} + \frac{2}{x^2+4}\right)$$

$$\frac{1}{16}\left(\frac{2}{4+x^2} - \frac{4x^2}{(x^2+4)^2} + \frac{2}{x^2+4}\right)$$

$$\frac{1}{16}\left(\frac{4(x^2+4)}{(4+x^2)^2} - \frac{4x^2}{(x^2+4)^2}\right)$$

$$\frac{1}{16}\left(\frac{4(4)}{(4+x^2)^2}\right). \checkmark$$