PRINT Your Name:____________________

Quiz 13 — November 25, 2013 – Section 1 – 3:30 – 4:20

Remove everything from your desk except a pencil or pen.

Write in complete sentences. Explain your work!
The quiz is worth 5 points.

For which x does the power series $f(x) = \sum\limits_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$ converge? **Explain your work very carefully. Write in complete sentences.**

Answer: We apply the ratio test. We see that

$$\rho = \lim_{\rho\to 0}\left|\frac{a_n}{a_{n-1}}\right| = \lim_{\rho\to 0}\left|\frac{\frac{(4x+1)^n}{n^2}}{\frac{(4x+1)^{n-1}}{(n-1)^2}}\right| = \lim_{\rho\to 0}\frac{|4x+1|^n}{n^2}\frac{(n-1)^2}{|4x+1|^{n-1}}$$

$$= \lim_{\rho\to 0}|4x+1|(1-\tfrac{1}{n})^2 = |4x+1|.$$

If $\rho < 1$, then $f(x)$ converges. If $1 < \rho$, then $f(x)$ diverges. We see that $\rho < 1$, when $-1 < 4x+1 < 1$, and this happens when $-2 < 4x < 0$ and this happens when $\frac{-1}{2} < x < 0$.

If $1 < \rho$, then $f(x)$ diverges. There are two ways to have $1 < \rho$; namely, if $1 < 4x+1$, then $1 < \rho$; also, if $4x+1 < -1$, then $1 < \rho$. In other words, if $0 < x$ or if $x < -\frac{1}{2}$, then $1 < \rho$.

We see that $f(0) = \sum\limits_{n=1}^{\infty}\frac{1}{n^2}$, which is the p-series with $p = 2 > 1$. Thus, $f(2)$ converges.

We see that $f(\frac{-1}{2}) = \sum\limits_{n=1}^{\infty}\frac{(-1)^n}{n^2}$. We use the Absolute Convergence Test. The series $\sum\limits_{n=1}^{\infty}|\frac{(-1)^n}{n^2}|$ is the same as $f(0)$, which converges. Thus, $\sum\limits_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ (which is $f(\frac{-1}{2})$) also converges.

We conclude that

$$\boxed{f(x) \text{ converges for } \frac{-1}{2} \le x \le 0, \text{ and diverges everywhere else.}}$$