PRINT Your Name:____________________

Quiz 10 — October 31, 2012 – Section 9 – 10:10 – 11:00

Remove everything from your desk except a pencil or pen.

Write in complete sentences.
The quiz is worth 5 points.

Find all values of x for which the series $\sum\limits_{n=0}^{\infty} \frac{\cos^n x}{2^n}$ converges. Find the sum of the series for those values of x.

Answer: The series $\sum\limits_{n=0}^{\infty} \frac{\cos^n x}{2^n}$ is a geometric series with initial term $a = \frac{\cos^0 x}{2^0} = 1$ and ratio $r = \frac{\cos x}{2}$. We notice that $-1 \le \cos x \le 1$; hence, $\frac{-1}{2} \le \frac{\cos x}{2} \le \frac{1}{2}$ and $-1 < r < 1$. Thus the series $\sum\limits_{n=0}^{\infty} \frac{\cos^n x}{2^n}$ converges for all values of x and the sum of the series is $\frac{a}{1-r} = \frac{1}{1-\frac{\cos x}{2}} = \frac{2}{2-\cos x}$.

$$\boxed{\text{The series } \sum_{n=0}^{\infty} \frac{\cos^n x}{2^n} \text{ converges to } \frac{2}{2-\cos x} \text{ for all } x.}$$