PRINT Your Name:____________________

Quiz 10 — March 23, 2012 – Section 8 – 10:10 – 11:00

Remove everything from your desk except a pencil or pen.

Write in complete sentences.
The quiz is worth 5 points.

Does the series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$ converge? Explain your work very thoroughly. Write in complete sentences.

Answer: Compare the given series to the convergent p-series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, where $p = 2 > 1$. We see that every term in both series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is positive. We also see that

$$\frac{\cos^2 n}{n^2+1} \le \frac{1}{n^2+1}$$

because $\cos^2 n \le 1$ and

$$\frac{1}{n^2+1} \le \frac{1}{n^2}$$

because the denominator is larger on the left. Thus,

$$\frac{\cos^2 n}{n^2+1} \le \frac{1}{n^2}.$$

The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges; thus $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$ also converges by the first part of the comparison test.