PRINT Your Name:_____

Remove everything from your desk except a pencil or pen.

Write in complete sentences. Explain your work!
The quiz is worth 5 points.

Estimate the error which results if $\sum\limits_{n=1}^{10} \frac{1}{n^4}$ is used to approximate $\sum\limits_{n=1}^{\infty} \frac{1}{n^4}$. Write a coherent and correct explanation for your estimate. Draw a meaningful picture.

Answer: The distance between $\sum\limits_{n=1}^{\infty} \frac{1}{n^4}$ and $\sum\limits_{n=1}^{10} \frac{1}{n^4}$ is equal to

$$\left| \sum_{n=1}^{\infty} \frac{1}{n^4} - \sum_{n=1}^{10} \frac{1}{n^4} \right| = \sum_{n=11}^{\infty} \frac{1}{n^4}$$

$$= \text{the area inside the boxes} \leq \text{the area under the curve} = \int_{10}^{\infty} \frac{1}{x^4}dx$$

$$= \lim_{b \to \infty} \int_{10}^{b} \frac{1}{x^4}dx = \lim_{b \to \infty} \frac{1}{-3x^3}\bigg|_{10}^{b} = \lim_{b \to \infty} \frac{1}{-3b^3} + \frac{1}{3(10^3)} = \frac{1}{3000}.$$

We conclude that

$$\boxed{\sum_{n=1}^{10} \frac{1}{n^4} \text{approximates} \sum_{n=1}^{\infty} \frac{1}{n^4} \text{ with an error of at most } \frac{1}{3000}.}$$