Final Exam, Math 142, Fall 1998

PRINT Your Name:__________________________________

There are 19 problems on 8 pages. Problems 1 and 2 are worth 7 points each. Each of the other problems is worth 8 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **Check** your answer whenever posible. **No Calculators.**

1. Find the Taylor polynomial of degree three, $P_3(x)$, for $f(x) = \ln x$ about $a = 1$.

2. Find an upper bound for the difference between $f(x)$ and $P_3(x)$ (from problem 1) when $|x-1| \le \frac{1}{10}$.

3. Sketch the region in the first quadrant that is inside $r = 3 + 3\cos\theta$ and outside $r = 3 + 3\sin\theta$, and find its area.

4. Let $f(x) = \dfrac{x}{e^{3x}}$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? Find all local extreme points of $y = f(x)$ and all points of inflection of $y = f(x)$. Graph $y = f(x)$.

5. Find $\displaystyle\int_{-1}^{2} \frac{1}{x^2}\,dx$.

6. Find $\displaystyle\int xe^{3x}\,dx$.

7. Find $\displaystyle\int \frac{5x^2 - x - 1}{x^3 - x^2}\,dx$.

8. Find $\displaystyle\int \sin^6 x \cos^3 x\,dx$.

9. Find $\displaystyle\int \frac{1}{\sqrt{x^2 + 4x + 5}}\,dx$.

10. Find $\displaystyle\lim_{x\to 0} \frac{x^4}{\cos x - 1 + \frac{x^2}{2}}$.

11. Find $\displaystyle\lim_{x\to\infty} \left(1 - \frac{1}{2x}\right)^x$.

12. Where does the power series function $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(x-5)^n}{2^n}$ converge?

13. What familiar function is equal to $f(x) = 1 + x^2 + \dfrac{x^4}{2!} + \dfrac{x^6}{3!} + \dfrac{x^8}{4!} + \dfrac{x^{10}}{5!} + \dfrac{x^{12}}{6!} + \ldots$?

14. Does $\displaystyle\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{(3)^{\frac{1}{n}}}$ converge? (Explain your answer.)

15. Does $\displaystyle\sum_{n=2}^{\infty}\frac{\sin n}{n^3}$ converge? (Explain your answer.)

16. Does $\displaystyle\sum_{n=2}^{\infty}\frac{2^n+3^n}{4^n}$ converge? (Explain your answer.)

17. Newton's law of cooling states that the rate at which an object cools is proportional to the difference in temperature between the object and the surrounding medium. Thus, if an object is taken from an oven at 350° F and left to cool in a room at 70° F, then its temperature T after t hours will satisfy the differential equation

$$\frac{dT}{dt}=k(T-70).$$

If the temperature fell to 250° F after one hour, what will it be after 3 hours? (You may leave " ln " in your answer.)

18. Find the derivative of $y=\ln(e^{2x}+\sqrt{2x^2+3})$.

19. Approximate $\displaystyle\int_0^1 \cos(x^2)\,dx$ with an error less than 10^{-3} .