Math 142, Exam 3, Fall 2009

Use my paper. **Please turn the problems in order. Please leave 1 square inch in the upper left hand corner for the staple.**

The exam is worth 50 points. There are 9 problems. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

1. (5 points) Find the limit of the sequence whose n^{th} term is $a_n = n\sin\left(\frac{3}{n}\right)$.

2. (5 points) Does the series $\displaystyle\sum_{n=1}^{\infty}\left(1-\frac{2}{n}\right)^n$ converge? Justify your answer.

3. (5 points) Consider the following sequence of numbers: $a_2 = (1-\frac{1}{4})$, $a_3 = (1-\frac{1}{4})(1-\frac{1}{9})$, $a_4 = (1-\frac{1}{4})(1-\frac{1}{9})(1-\frac{1}{16})$, $\ldots$, $a_n = (1-\frac{1}{4})(1-\frac{1}{9})(1-\frac{1}{16})\ldots(1-\frac{1}{n^2})$, $\ldots$. Does this infinite sequence converge? Justify your answer.

4. (5 points) Does the series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}$ converge? Justify your answer.

5. (6 points) Does the series $\displaystyle\sum_{n=1}^{\infty}\frac{2\sqrt{n}}{n^2+1}$ converge? Justify your answer.

6. (6 points) Where does $f(x) = \displaystyle\sum_{n=1}^{\infty}\frac{(x-2)^n}{n3^n}$ converge? Justify your answer.

7. (6 points) Approximate $e^{\frac{-1}{10}}$ with an error at most 10^{-3}. Explain what you are doing.

8. (6 points) Approximate $\displaystyle\sum_{n=10}^{\infty}\frac{1}{n^2}$. Your approximation should be "close to" but more than the actual value. Explain what you are doing.

9. (6 points) A ball is dropped from a height of 100 feet. Each time it bounces, it rebounds to $\frac{4}{5}$ its previous height. Find the total distance it travels before coming to rest.