Math 142, Final Exam, Fall 2002

Name _____

There are 16 problems on 8 pages. Problems 1 through 6 are worth 10 points each. Problems 7 through 16 are worth 9 points each.

✓ **Check your answer**, whenever possible.

$\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

I will post an answer key on my web site shortly after the exam is finished. If I know your e-address, I will e-mail your grade to you as soon as it is ready; otherwise, get your grade from VIP.

1. Find $\displaystyle\int x \ln x \, dx$.

2. Find $\displaystyle\int \frac{\ln x}{x} \, dx$.

3. Find $\displaystyle\int_{2}^{5} \frac{dx}{(x-3)^4}$.

4. Find $\displaystyle\int \sin^2 x \cos^3 x \, dx$.

5. Approximate $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$, with an error of at most $\frac{1}{100}$. Explain what you are doing!

6. Approximate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$, with an error of at most $\frac{1}{100}$. Explain what you are doing!

7. Approximate $e^{\frac{1}{10}}$, with an error of at most $\frac{1}{100}$. Explain what you are doing!

8. Find the limit of the sequence whose n^{th} term is
$a_n = \left(\frac{n-2}{n}\right)^{3n}$.

9. Find $\displaystyle\int \frac{3x^3 - x^2 + x}{(x^2 + 1)^2} dx$.

10. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$ converge? Justify your answer.

11. Find $\displaystyle\lim_{x \to 0} \frac{\cos x^3 - 1 + \frac{x^6}{2}}{x^{12}}$.

12. Find $\displaystyle\int \sqrt{1 + x^2}\, dx$.

13. Graph $r = \sin 2\theta$.

14. Where does $\displaystyle f(x) = \sum_{n=1}^{\infty} \frac{(x + 2)^n}{4^n n^2}$ converge? Justify your answer.

15. Solve the initial value problem $\frac{dy}{dx} - \frac{y}{x} = 3x^2$ and $y(1) = 3$.

16. Newton's law of cooling states that the rate at which an object cools is proportional to the difference in temperature between the object and the surrounding medium. Thus, if an object is taken from an oven at $300°$ F and left to cool in a room at $80°$ F, then its temperature T after t hours will satisfy the differential equation

$$\frac{dT}{dt} = k(T - 80).$$

If the temperature fell to $200°$ F after one hour, what will it be after 3 hours? (You may leave "ln" in your answer.)