Math 142, Exam 4, Fall 2002

Name _____

There are 11 problems on 6 pages. Problem 1 is worth 10 points. Each of the other problems is worth 9 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

1. Find the limit of the sequence whose n^{th} term is $a_n = n \sin\left(\frac{3}{n}\right)$.

2. Does the series $\sum_{n=1}^{\infty} \left(1 - \frac{2}{n}\right)^n$ converge? Justify your answer.

3. Consider the following sequence of numbers: $a_2 = (1 - \frac{1}{4})$, $a_3 = (1 - \frac{1}{4})(1 - \frac{1}{9})$, $a_4 = (1 - \frac{1}{4})(1 - \frac{1}{9})(1 - \frac{1}{16})$, $\ldots$, $a_n = (1 - \frac{1}{4})(1 - \frac{1}{9})(1 - \frac{1}{16}) \ldots (1 - \frac{1}{n^2})$, $\ldots$. Does this infinite sequence converge? Justify your answer.

4. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converge? Justify your answer.

5. Does the series $\sum_{n=1}^{\infty} \frac{2\sqrt{n}}{n^2 + 1}$ converge? Justify your answer.

6. Where does $f(x) = \sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$ converge? Justify your answer.

7. Find $\lim_{x \to 0} \frac{\sin(x^2) - x^2 + \frac{x^6}{6}}{x^{10}}$.

8. Which familair function is equal to $f(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{3!} + \frac{x^6}{4!} + \frac{x^8}{5!} + \frac{x^{10}}{6!} + \ldots$? Explain.

9. Approximate $e^{\frac{-1}{10}}$ with an error at most 10^{-3}. Explain what you are doing.

10. Approximate $\sum_{n=10}^{\infty} \frac{1}{n^2}$. Your approximation should be "close to" but more than the actual value. Explain what you are doing.

11. A ball is dropped from a height of 100 feet. Each time it bounces, it rebounds to $\frac{4}{5}$ its previous height. Find the total distance it travels before coming to rest.