Math 142, Fall 2001, Final Exam

PRINT Your Name:_____

Get your course grade from **TIPS/VIP** late on Tuesday or later. There are 20 problems on 10 pages. The exam is worth 150 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS! CHECK** your answer whenever possible.

1. (8 points) Find $\displaystyle\int \sin^5 x \, dx$. (Be sure to check your answer.)

2. (8 points) Find $\displaystyle\int \sin^4 x \, dx$.

3. (8 points) Where does the function $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(x-7)^n}{n \cdot 3^n}$ converge? Justify your answer.

4. (8 points) Solve the differential equation $\frac{dy}{dt} = 6y$ with the initial condition $y(1) = 4$. Check your answer.

5. (8 points) Does $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{2}{n}\right)^n$ converge? Justify your answer.

6. (8 points) Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$ converge? Justify your answer.

7. (8 points) Does $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$ converge? Justify your answer.

8. (8 points) Does $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^5}$ converge? Justify your answer.

9. (8 points) Suppose that the government pumps an extra \$1 billion into the economy. Assume that each business and individual saves 25% of its income and spends the rest, so that of the initial \$1 billion, 75% is respent by individuals and businesses. Of that amount, 75% is spent, and so forth. What is the total increase in spending due to the government action?

10. (8 points) Find the general solution of $\frac{dy}{dx} - \frac{y}{x} = 3x^3$. Check your answer.

11. (7 points) Find $\displaystyle\int_{-3}^{1} \frac{1}{x^2} dx$.

12. (7 points) Find $\displaystyle\int \sqrt{1 + x^2} \, dx$. (Be sure to check your answer.)

13. (7 points) Find $\displaystyle\int \frac{x}{\sqrt{9 - 16x^2}}\, dx$. (Be sure to check your answer.)

14. (7 points) Find $\displaystyle\int \frac{1}{\sqrt{9 - 16x^2}}\, dx$. (Be sure to check your answer.)

15. (7 points) Find $\displaystyle\int \ln x \, dx$. (Be sure to check your answer.)

16. (7 points) Graph $r = 2 - 4\sin\theta$. Find the area inside the inner loop of this graph.

17. (7 points) Let $f(x) = x\ln x$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? Find the local maxima, local minima, and points of inflection of $y = f(x)$. Graph $y = f(x)$.

18. (7 points) Find $\displaystyle\int \frac{3 + 2x + 7x^2 - 3x^3}{x^4 + x^2}\, dx$. (Be sure to check your answer.)

19. (7 points) Which familiar function is equal to $\dfrac{1}{2!} + \dfrac{x}{3!} + \dfrac{x^2}{4!} + \dfrac{x^3}{5!} + \dfrac{x^4}{6!} + \dfrac{x^5}{7!} + \ldots$? Justify your answer.

20. (7 points) Find the third Taylor polynomial $P_3(x)$ for $f(x) = \ln x$ about $a = 1$. Estimate the error that is introduced if $f(x)$ is approximated by $P_3(x)$ for $.8 \le x \le 1.2$.