

### Fall 2001, Exam 3, Math 142

PRINT Your Name: \_\_\_\_\_

There are 11 problems on 6 pages. Problem 1 is worth 10 points. Each of the other problems is worth 9 points. SHOW your work. **CIRCLE** your answer. **NO CALCULATORS!** CHECK your answer whenever possible. If you want to pick up your exam before Monday, write a short note to that effect on the top of this page and I will leave your exam outside my office door, before I go home tonight.

1. Find  $\int \sin^3 x dx$ . Check your answer.
2. Find  $\int \cos^4 x dx$ .
3. Find the general solution of  $\frac{dy}{dx} - 3y = xe^{3x}$ . Check your answer.
4. Find  $\int e^{-x} \cos x dx$ . Check your answer.
5. Find  $\int \frac{6x^2 - 3x + 1}{(4x+1)(x^2+1)} dx$ . Check your answer.
6. Consider the sequence whose  $n^{\text{th}}$  term is  $a_n = \left(\frac{n-3}{n}\right)^n$ . Find the limit of this sequence.
7. Find  $\lim_{x \rightarrow 1^-} \frac{x-1}{\arctan x}$ .
8. Find  $\int_{-3}^1 \frac{1}{x^2} dx$ .
9. Find  $\int_1^{\infty} \frac{x}{e^x} dx$ .
10. Consider the series  $\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$ . Find a closed formula for the partial sum  $s_n = \sum_{k=1}^n \ln\left(\frac{k}{k+1}\right)$ . (In other words, I want you to find a formula which is equal to  $s_n$ . Your formula is not allowed to contain any “dots” or any summation signs.) Does the original series converge or diverge? Find the limit of the series, if possible.
11. A ball is dropped from a height of 100 feet. Each time it hits the floor, it rebounds to  $\frac{2}{3}$  its previous height. Find the total distance the ball travels before coming to rest.