Exam 1, Fall 2001, Math 142

PRINT Your Name:_____

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS! CHECK** your answer whenever possible.

1. Find $\int \frac{e^x}{e^x+1}dx$. Check your answer.

2. Find $\int \frac{2^x}{\sqrt{2^x+1}}dx$. Check your answer.

3. If $y = x^x$, then find $\frac{dy}{dx}$.

4. Find the volume of the solid generated by revolving the region bounded by $y = e^x$, the $x-$ axis, the $y-$ axis, and $x = 1$ about the x-axis.

5. Let $f(x) = x^2 + 2x$ for $x \le -1$. Find $f^{-1}(x)$. What is the domain of $f^{-1}(x)$? Verify that $f(f^{-1}(x)) = x$ for all x in the domain of $f^{-1}(x)$. Verify that $f^{-1}(f(x)) = x$ for all x in the domain of $f(x)$. Graph $y = f(x)$. Graph $y = f^{-1}(x)$.

6. If $y = e^{\frac{1}{x^2}} + \frac{1}{e^{x^2}}$, then find $\frac{dy}{dx}$.

7. Let $f(x) = xe^{\frac{x}{2}}$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? Find the local maxima, local minima, and points of inflection of $y = f(x)$. Find all vertical and horizontal asymptotes of $y = f(x)$. Graph $y = f(x)$.

8. Solve the differential equation $\frac{dy}{dt} = -6y$ with the initial condition $y(1) = 4$. Check your answer.

9. Solve for x: $\log_2 (x + 3) - \log_2 x = 2$. Check your answer.

10. Newton's law of cooling states that the rate at which an object cools is proportional to the difference in temperature between the object and the surrounding medium. Thus, if an object is taken from an oven at $400°$ F and left to cool in a room at $60°$ F, then its temperature T after t hours will satisfy the differential equation

$$\frac{dT}{dt} = k(T - 60).$$

If the temperature fell to $300°$ F after one hour, what will it be after 4 hours? (You may leave "ln" in your answer.)