Final Exam, Math 142, Fall 2000

PRINT Your Name:________________________________

Get your course grade from **TIPS/VIP** late on Tuesday or later.

There are 20 problems on 8 pages. Problems 1 to 10 are worth 8 points each. Each of the other problems is worth 7 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **Check** your answer whenever posible. **No Calculators.**

1. Find $\displaystyle\int \sin^3 x \cos^2 x \, dx$.

2. Find $\displaystyle\int \cot x \, dx$.

3. Find $\displaystyle\int x e^{3x} \, dx$.

4. Find the Taylor polynomial of degree three, $P_3(x)$, for $f(x) = \ln(1-x)$ about $a = 0$.

5. Find an upper bound for the difference between $f(x)$ and $P_3(x)$ (from problem 4) when $|x| \le \frac{1}{100}$.

6. Find $\displaystyle\int \frac{dx}{x^2 + 4x + 13}$.

7. Find $\displaystyle\int \frac{-x^2 + 5x - 2}{(x-1)(x^2+1)} \, dx$.

8. Find $\displaystyle\int_0^\infty \frac{x}{(x^2+2)^2} \, dx$.

9. Find $\displaystyle\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.

10. Find $\displaystyle\lim_{x \to \pi} \frac{1 - \cos x}{x^2}$.

11. Let $f(x) = x^3 e^{\sin(x^2)}$. Find $f'(x)$.

12. Let $f(x) = \frac{17}{\arcsin x}$. Find $f'(x)$.

13. What infinite power series is equal to $f(x) = \sin(x^2)$ about $a = 0$? (A different way to phrase the same problem is, "Find the Taylor series for $f(x) = \sin(x^2)$ about $a = 0$ ".)

14. Find a closed formula for $\displaystyle\sum_{k=3}^{75} 3^k = 3^3 + 3^4 + 3^5 + 3^6 + \cdots + 3^{75}$. (Your answer should not have any summation signs and should not have any dots.)

15. Does $\sum_{n=2}^{\infty} \frac{3}{n^2+1}$ converge? (Explain your answer.)

16. Does $\sum_{n=2}^{\infty} \frac{n}{3^n}$ converge? (Explain your answer.)

17. Does $\sum_{n=3}^{\infty} \left[\frac{3}{n} - \frac{3}{n+1}\right]$ converge? (Explain your answer.)

18. Does $\sum_{n=2}^{\infty} \frac{8}{n \ln n}$ converge? (Explain your answer.)

19. Newton's law of cooling states that the rate at which an object cools is proportional to the difference in temperature between the object and the surrounding medium. Thus, if an object is taken from an oven at 300° F and left to cool in a room at 75° F, then its temperature T after t hours will satisfy the differential equation

$$\frac{dT}{dt} = k(T - 75).$$

If the temperature fell to 200° F after one hour, what will the temperature be after 4 hours? (You may leave "ln" in your answer.)

20. Where does the power series function $f(x) = \sum_{n=1}^{\infty} \frac{(x+2)^n}{3^n}$ converge?