Math 142, Fall 2000, Exam 4

PRINT Your Name:_____

There are 11 problems on 6 pages. Problem 1 is worth 10 points. Each of the other problems is worth 9 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

1. Find the Taylor polynomial of degree six, $P_6(x)$, for $f(x) = \sin x$ about $a = 0$.

2. Use your answer to problem 1 to estimate $\displaystyle\int_0^1 \frac{\sin x}{x} dx$. How good is your estimate? Explain.

3. Use Simpson's rule to estimate the area of the following shape. All measurements are in feet.

4. Does the series $\displaystyle\sum_{k=1}^{\infty} \left(1 - \frac{1}{k}\right)^k$ converge? Justify your answer. Find the sum of the series if you can.

5. Does the series $\displaystyle\sum_{k=1}^{\infty} \frac{5}{k+3}$ converge? Justify your answer. Find the sum of the series if you can.

6. Does the series $\displaystyle\sum_{k=1}^{\infty} \frac{3^{k+1}}{2^{k-1}}$ converge? Justify your answer. Find the sum of the series if you can.

7. A ball is dropped from the height of 10 feet. Each time it hits the floor it rebounds to $\frac{2}{3}$ its previous height. Find the total distance it travels.

8. Consider the series $\displaystyle\sum_{k=4}^{\infty} \frac{1}{3^k}$. Give a closed formula for the partial sum $s_n = \displaystyle\sum_{k=4}^{n} \frac{1}{3^k}$. Does the series converge? If so, what is the sum of the series?

9. Consider the series $\displaystyle\sum_{k=4}^{\infty} \frac{1}{k} - \frac{1}{k+1}$. Give a closed formula for the partial sum $s_n = \displaystyle\sum_{k=4}^{n} \frac{1}{k} - \frac{1}{k+1}$. Does the series converge? If so, what is the sum of the series?

10. Find $\displaystyle\int_0^{\infty} \frac{dx}{1+x^2}$.

11. Find $\displaystyle\lim_{x \to 0} \frac{\cos x - 1 + \frac{x^2}{2}}{x^4}$.