Math 142, Final Exam, Spring 2011

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 100 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

No Calculators, Cell phones, or computers.

1. Find $\int x \ln x dx$. **Check your answer.**

2. Find $\int \cos^5 x \sin^2 x dx$. **Check your answer.**

3. Find $\int \cos^4 x dx$.

4. Find $\int \frac{x}{x^2-2x+5} dx$. **Check your answer.**

5. Find $\int \frac{x+1}{(x-1)^2} dx$. **Check your answer.**

6. Find $\int \frac{4x-7}{x^2-3x+2} dx$. **Check your answer.**

7. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, the x-axis, $x = 1$, and $x = 2$ about the line $y = 5$. **Write in complete sentences. Make your work coherent.**

8. Consider a solid with base bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. Each cross section of the solid perpendicular to the y-axis is a square. Find the volume of the solid. **Write in complete sentences. Make your work coherent.**

9. Consider the sequence $\{a_n\}$, with $a_1 = 50$, and, for $n \geq 2$, $a_n = \sqrt{2 + a_{n-1}}$. Prove that the sequence $\{a_n\}$ converges. Find the limit of the sequence. **Write in complete sentences. Make your work coherent.**

10. Write $4.3214214\overline{214}\ldots$ as the ratio of two integers. Explain what you are doing. **Write in complete sentences. Make your work coherent.**

There are more questions on the other side.

11. Does the series $\sum_{n=1}^{\infty} \sin(\frac{1}{n})$ converge? **Justify your answer very thoroughly. Use complete sentences.**

12. Does the series $\sum_{n=1}^{\infty} n^2 e^{-n}$ converge? **Justify your answer very thoroughly. Use complete sentences.**

13. Where does $f(x) = \sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$ converge? **Justify your answer very thoroughly. Use complete sentences.**

14. Find the Taylor polynomial $T_2(x)$ for $f(x) = \ln x$ about $a = 1$.

15. Keep $f(x)$, a, and $T_2(x)$ as in problem 14. Give an upper bound on the error that is introduced if $T_2(x)$ is used in place of $f(x)$ for $.9 \leq x \leq 1.1$. **Justify your answer very thoroughly. Use complete sentences.**

16. Approximate $\int_0^{\frac{1}{10}} \sin(x^2)dx$ with an error at most $\frac{1}{10^7}$. **Justify your answer very thoroughly. Use complete sentences.**

17. Compute $\lim_{x\to 0} \frac{e^{x^2}-1-x^2-\frac{x^4}{2}}{x^6}$. **Justify your answer very thoroughly. Use complete sentences.**

18. Find the sum of the series $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \dots$. **Justify your answer very thoroughly. Use complete sentences.**