<p style="text-align:center">**Math 142,   Final Exam,   Spring 2012**</p>

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 100 points. There are **18** problems on **2** sides.

No Calculators or Cell phones. Write in complete sentences. Explain what you are doing VERY thoroughly.

1. (6 points) Find $\int \sin^5 x \cos^2 x\, dx$. **Check your answer.**

2. (6 points) Find $\int \sin^2 x \cos^2 x\, dx$.

3. (6 points) Find $\int \tan^5 x \sec x\, dx$. **Check your answer.**

4. (6 points) Find $\int \frac{1}{3x^2+6x+7}\, dx$. **Check your answer.**

5. (6 points) Find $\int \frac{2x^3+9x^2+10x+1}{(x+1)^2(x^2+1)}\, dx$. **Check your answer.**

6. (6 points) Find $\int x \ln x\, dx$. **Check your answer.**

7. (6 points) Give an upper bound for the difference between $\sum\limits_{n=1}^{\infty} \frac{1}{n^4}$ and $\sum\limits_{n=1}^{10} \frac{1}{n^4}$.
   Be sure to explain what you are doing. Write in complete sentences. You must draw a meaningful picture.

8. (6 points) Compute $\lim\limits_{x\to 0} \frac{\sin(x^2)-x^2+\frac{x^6}{6}-\frac{x^{10}}{120}}{x^{14}}$. **Be sure to explain what you are doing. Write in complete sentences.**

9. (6 points) Estimate $\int_0^{1/10} \frac{\sin(x^3)}{x}\, dx$ with an error of at most 10^{-10}. **Be sure to explain what you are doing.**

10. (6 points) What familiar function is equal to $2 \cdot 1 + 3 \cdot 2x + 4 \cdot 3x^2 + 5 \cdot 4x^3 + 6 \cdot 5x^4 + \ldots$?

   PLEASE TURN OVER.

11. (5 points) Let $f(x) = \sqrt{x}$. Find the second Taylor polynomial $T_2(x)$ for $f(x)$ about $a = 1$.

12. (5 points) Keep the notation of problem 11. Give an upper bound for the distance between $f(x)$ and $T_2(x)$ for $.9 < x < 1.1$. **Be sure to explain what you are doing. Write in complete sentences.**

13. (5 points) Where does the power series $f(x) = \sum_{n=1}^{\infty} \frac{(x-6)^n}{\sqrt{n}3^n}$ converge? **Be sure to explain what you are doing. Write in complete sentences.**

14. (5 points) Does $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ converge? **Justify your answer very thoroughly. Write in complete sentences.**

15. (5 points) Does the series $\sum_{n=1}^{\infty} \frac{\sqrt{n-1}}{n^3+2n^2+5}$ converge? **Justify your answer very thoroughly. Write in complete sentences.**

16. (5 points) Consider the sequence defined by $a_1 = 2$ and $a_{n+1} = \frac{1}{4-a_n}$. **Justify your answer very thoroughly. Write in complete sentences.**
    (a) Prove that $0 < a_n \leq 2$ for all positive integers n.
    (b) Prove that $a_{n+1} \leq a_n$ for all positive integers n.
    (c) State the Completeness Axiom and draw a conclusion about the sequence $\{a_n\}$ from the Completeness Axiom.
    (d) Find the limit of the sequence $\{a_n\}$.

17. (5 points) Consider the region bounded by $y = x^2$ and $x = y - 6$. Revolve the region about $x = -4$. Find the volume of the resulting solid. **Be sure to explain what you are doing. Write in complete sentences. You must draw a meaningful picture.**

18. (5 points) Consider the solid whose base is bounded by $\frac{x^2}{9} + \frac{y^2}{16} = 1$ in the xy-plane. Each cross section of the solid perpendicular to the x-axis and perpendicular to the base is an equilateral triangle. Find the volume of the solid. **Be sure to explain what you are doing. Write in complete sentences. You must draw a meaningful picture.**